Progetto di ricerca correlato all'assegno: Birational Geometry, Moduli spaces and their invariants.

The modern study of moduli spaces, and their invariants, involves sophisticated techniques that were created to analyze geometric objects whose structure is extremely rich, as well as quite mysterious. This project combines several of these powerful machineries to unveil some of the undiscovered behaviors of many interesting moduli spaces and their birational properties in algebraic geometry.

The project will focus on the research topics of our PRIN group in algebraic geometry. We will work on degenerations and smoothings of algebraic varieties, their compactifications and stability. Some of the main topics are:

1. **Fano and toric varieties**. Fano varieties are the algebraic varieties `with positive curvature', whereas toric varieties are algebraic varieties which can be constructed from objects in discrete geometry such as polytopes. We aim to study Fano varieties, and in particular toric ones, from the point of view of deformation theory (e.g. existence of smoothings), K-stability (i.e. existence of Kaehler-Einstein metrics), birational geometry, in order to study moduli spaces of Fano varieties.

2. **Curves in Surfaces and threefolds.** We aim to study families of surfaces and threefolds and families of curves in surfaces, such as K3 surfaces, or surfaces of general type (varieties with zero and negative curvature respectively) and threefolds and related topics, such as deformations of singularities, compactifications of moduli spaces, Noether-Lefschetz theory and Brill-Noether theory.

3. **Fibrations.** We aim to study fibration structures on algebraic varieties, in particular elliptic fibrations, fibrations with general fibre a curve of genus two or a K3 surface. This has important implications in birational classifications of higher dimensional varieties and connections with mathematical physics.

These three topics are deeply intertwined with each other: for example, the structure of elliptic and K3 fibrations in Calabi-Yau varieties (varieties with zero curvature) which are anticanonical divisors in toric Fano varieties is ubiquitous; the study of elliptic fibrations and of deformations of (toric) Fano varieties is central in "mirror symmetry" and theoretical physics.

Activity Plan:

The activity will take place at the Department of Mathematics, University of Bologna, within the funded project PRIN_GRASSI 2022, "*GEOMETRY OF ALGEBRAIC STRUCTURES: MODULI, INVARIANTS, DEFORMATIONS",* Codice Progetto: 2022BTA242, PI Ugo Bruzzo.

The fellow is expected to give regular research seminars on the project. In addition, the fellow will participate to group meetings where the research group will discuss recent publications and seminar announcements. The fellow will be present a summary of the activities at the end of the fellowship period.